# THE SELBERG INTEGRAL AND THE PAIR-CORRELATION FUNCTION FOR THE ZEROS OF THE RIEMANN ZETA-FUNCTION

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Abstract: Let

$$J(X,\theta)=\int_X^{2X}|\psi(t)-\psi(t-\theta t)-\theta t|^2\,\mathrm{d}t$$

denote the "variance" of the primes in short intervals, where $\theta\in[0,1]$ and

$$F(X,T)=\sum_{\gamma_1,\gamma_2\in[0,T]}\frac{4X^{\mathrm{i}(\gamma_1-\gamma_2)}}{4+(\gamma_1-\gamma_2)^2}$$

denote Montgomery's pair-correlation function for the zeros of the Riemann zeta-function. We extend results of Goldston & Montgomery, Montgomery & Soundararajan and Chan, providing more accurate relations between hypothetical two-term asymptotic formulas for $J$ and $F$ (see [1]). We also propose a new, more general pair-correlation function and the corresponding version of the Selberg integral, and discuss their relevance to the distribution of prime numbers in short intervals (see [2], [3]).

This is a joint work with Alessandro Languasco and Alberto Perelli.

[1] A. Languasco, A. Perelli, and A. Zaccagnini, *Explicit relations between pair correlation of zeros and primes in short intervals*, J. Math. Anal. Appl. **394** (2012), 761–771.

[2] A. Languasco, A. Perelli, and A. Zaccagnini, *An extension of the pair-correlation conjecture and applications*, Math. Res. Lett. **23** (2016), no. 1, 201–220.

[3] A. Languasco, A. Perelli, and A. Zaccagnini, *An extended pair-correlation conjecture and primes in short intervals*, Trans. Amer. Math. Soc. **369** (2017), no. 6, 4235–4250.