DIVISOR-SUM FIBERS

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Abstract: Let $s(\cdot)$ denote the sum-of-proper-divisors function, that is, $s(n) = \sum_{d|n,\ d<n} d$. Erdős–Granville–Pomerance–Spiro conjectured that, for any set $\mathcal{A}$ of asymptotic density zero, the preimage set $s^{-1}(\mathcal{A})$ also has density zero. We prove a weak form of this conjecture. In particular, we show that the EGPS conjecture holds for infinite sets with counting function $O(x^{\frac{1}{2}+\epsilon(x)})$. We also disprove a hypothesis from the same paper of EGPS by showing that for any positive numbers α and ϵ, there are integers n with arbitrarily many s-preimages lying between $\alpha(1-\epsilon)n$ and $\alpha(1+\epsilon)n$. This talk is based on joint work with Paul Pollack and Carl Pomerance.