ON THE LOGARITHMIC CHOWLA AND ELLIOTT CONJECTURES

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ABSTRACT: We consider k-point correlations of 1-bounded multiplicative functions $g_1, \dots, g_k$ with logarithmic averaging, that is, sums of the form

$$\frac{1}{\log x} \sum_{n \leq x} \frac{g_1(n+h_1) \cdots g_k(n+h_k)}{n},$$

where $h_1, \dots, h_k$ are distinct shifts. A conjecture of Elliott, in its weaker, logarithmic form, states that the quantity above is $o(1)$, unless all of the functions g_j "pretend to be" twisted Dirichlet characters $n \mapsto \chi_j(n) n^{it_j}$ in the sense of pretentious distance for multiplicative functions. This conjecture was confirmed by Tao in the two-point case $k = 2$.

We consider the case of general k-point correlations and show that the correlation average is $o(1)$ under certain additional non-pretentiousness assumptions. This in particular allows us to obtain the odd order cases of the logarithmic Chowla conjecture on correlations of the Liouville function.

This is joint work with Terence Tao.