LIMIT DISTRIBUTIONS FOR SOME SETS OF ADDITIVE FUNCTIONS

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Abstract: Let $x > 2$ and $f_x : \mathbb{N} \to \mathbb{R}$ be a set of strongly additive functions and let they take values 0 or 1 on primes. The limit behaviour of distributions

$$\frac{1}{[x]} \sum_{\substack{n \leqslant x \\ f(n)-\alpha(x)<u\beta(x)}} 1$$

was considered in the probabilistic number theory very often with different centering and normalizing functions $\alpha(x)$ and $\beta(x)$. In the books [1], [2], [3], and works cited there, one can find almost all classical results and their historical context.

An object of the talk is a weak convergence of distributions

$$\frac{1}{[x]} \sum_{\substack{n \leqslant x \\ f_{1x}(A(n))+f_{2x}(B(n))<u}} 1, \tag{1}$$

where $A(n), B(n)$ are two arithmetically interesting subsequences of positive integers. Some distributions can arise as limit distributions for (1) as $x \to \infty$, some ones can not. In the talk we will concentrate on the Poisson, binomial, and discrete uniform distributions. From the limit behaviour of (1), interesting conclusions on the common asymptotic properties and multiplicative structure of the sets $A(n), B(n)$ can be deduced. For instance,

$$\#\left\{p : \begin{array}{c} p \text{ prime}, p \leqslant x, p+1 \text{ and } p+2 \\ \text{have no prime factors in } (\log x, (\log x)^2) \end{array}\right\} \sim \frac{x}{4\log x}$$

as $x \to \infty$.

[1] P.D.T.A. Elliott, *Probabilistic Number Theory, I*, Springer-Verlag, New York, 1979.

[2] P.D.T.A. Elliott, *Probabilistic Number Theory, II*, Springer-Verlag, New York, 1980.

[3] J. Kubilius, *Probabilistic Methods in the Theory of Numbers* , Providence, Amer. Math. Soc. Translations of Math. Monographs, No **11** (1964).