# A LOGARITHMIC IMPROVEMENT IN THE BOMBIERI-VINOGRADOV THEOREM

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ABSTRACT: For integer number $a$ and $q \geq 1$, let

$$\psi(x; q, a) = \sum_{\substack{n \leq x \\ n \equiv a \pmod{q}}} \Lambda(n),$$

where $\Lambda(n)$ is the von Mangoldt function. The Bombieri-Vinogradov theorem is an estimate for the error terms in the prime number theorem for arithmetic progressions averaged over all $q$ up to $x^{1/2}$, or, rather almost all $q$ up to $x^{\frac{1}{2}}$.

**Theorem** (Bombieri-Vinogradov)**.** *Let $A$ be a given positive number and $Q \leq \frac{x^{1/2}}{(\log x)^B}$, where $B = B(A)$. Then*

$$\sum_{q \leq Q} \max_{2 \leq y \leq x} \max_{\substack{a \\ (a,q)=1}} \left| \psi(y, q, a) - \frac{y}{\varphi(q)} \right| \ll_A \frac{x}{(\log x)^A}.$$

The implied constant in this theorem is not effective, since we have to take care of characters associated with those $q$ that have small prime factors. At the same time, effective versions – in which the effect of an exceptional character is avoided in one way or another - have been known since the paper by Timofeev and Lenstra-Pomerance work on Gaussian periods.

We improve the best known to date result of Dress-Iwaniec-Tenenbaum, getting $(\log x)^2$ instead of $(\log x)^{\frac{5}{2}}$. We use a weighted form of Vaughan's identity, allowing a smooth truncation inside the procedure, and an estimate due to Barban-Vehov and Graham related to Selberg's sieve. We sketch the proof of effective and non-effective versions of the result. We also briefly discuss the possibility of getting the fully effective Bombieri-Vinogradov theorem for $q \leq x^{\frac{1}{2}-\varepsilon}$. The ineffectivity is avoided by applying Landau-Page results without using Siegel-Walfisz theorem.