

POWER SERIES WITH NUMBER THEORETIC COEFFICIENTS

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ABSTRACT: If $F(z) = \sum_{n \geq 1} a_n z^n$ is a power series, and ζ is a primitive q -th root of unity, then

$$\sum_{i=1}^q F(\zeta^i z) = q \sum_{q|n} a_n z^n.$$

This simple observation translates statements about the coefficients of a power series into statements about the function represented by the power series. We show that this translation can be applied in various different situations. First we consider the approximation of power series with number theoretic coefficients by rational and algebraic functions, improving on work by Banks, Luca and Shparlinski. Then we consider a question posed by Schwarz on the boundary behaviour of a power series with coefficients having some number theoretic meaning. Finally we deal with a problem posed by Kátai concerning additive functions which asymptotically satisfy a linear relation.