

## RECENT RESULTS ABOUT CLOSE PRIMES AND SOME CONJECTURES OF ERDŐS

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ABSTRACT: Over the last 90 years lot of efforts were made by excellent mathematicians (Hardy, Littlewood, Rankin, Erdős, Bombieri, Davenport and others) to understand the behavior of gaps between consecutive primes. The best result was still in 2004 the assertion that there are infinitely many prime gaps which are smaller than the respective average  $\log x$  by a factor 0.248... (Helmut Maier, 1988). Twelve years ago in joint work with Goldston and Yıldırım we succeeded to show this with an arbitrarily small positive constant  $c$  in place of 0.248.... Somewhat later we showed that there are infinitely many gaps less than the square-root of the average gap size. Simultaneously we showed that under the very deep unproved Elliott-Halberstam conjecture there are infinitely many gaps of size at most 16. In a joint work with Motohashi we proved that there are infinitely many bounded gaps if the Bombieri-Vinogradov theorem can be improved somewhat for smooth moduli. In 2013 Zhang succeeded in this way to show the existence of infinitely many bounded gaps. Eight months later simultaneously and independently Maynard and Tao showed that for an arbitrary  $k$  we have a chain of  $k$  consecutive primes in bounded intervals of length  $C(k)$ . While Zhang's work was able to show strong results about gaps between consecutive primes the Maynard-Tao method made able to study consecutive gaps between primes. This method opened the way to prove a number of old conjectures, many of them stated 60-70 years ago by Erdős. In this lecture we give a survey of such results. To mention just one example: Erdős, Pólya and Turán conjectured 60 years ago that if we consider for an arbitrary  $k$  a linear combination (with fixed real coefficients) of  $k$  consecutive prime gaps then the expression takes infinitely many positive and negative values as well if and only if the non-zero coefficients are not all of the same sign. Some special cases were proved by Erdős (some in joint work with Turán).