SOME PROBLEMS REGARDING CORRELATIONS OF MULTIPLICATIVE FUNCTIONS

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Abstract: Time permitting, we shall discuss three applications of the theory of correlations of multiplicative functions. This is all joint work with Oleksiy Klurman.

i) Given a multiplicative function $f : \mathbb{N} \to \{-1, +1\}$, we show that the 4-tuples

$$(f(n), f(n+d), f(n+2d), f(n+3d))$$

equidistribute among the 16 possible sign patterns $\boldsymbol{\epsilon} \in \{-1, +1\}^4$, for *almost all* d, except under certain circumstances that we can classify. This roughly demonstrates that Chowla's conjecture for length 4 sign patterns holds almost always, except under natural obstructions. We will also discuss the irregularities of distribution in some of those exceptional cases. This generalizes and refines work of Buttkewitz and Elsholtz.

ii) We show that any unimodular, completely multiplicative function $f : \mathbb{N} \to \mathbb{T}$ satisfies

$$\liminf_{n\to\infty} |f(n+1) - f(n)| = 0.$$

This proves a folklore conjecture due to Kátai, Ruzsa, Elliott and others. We will also discuss some related questions concerning gaps between consecutive values of multiplicative functions.

iii) We show that a completely multiplicative function f is a non-principal Dirichlet character if, and only if: a) f takes values in roots of unity of bounded order; b) f vanishes at only finitely many primes; c) f has bounded partial sums, i.e.,

$$\sum_{n\le x} f(n) = O(1).$$

This answers an old question of Chudakov.