

POLYNOMIAL PELL EQUATIONS
 $P(X)^2 - (X^{2M} + AX + B)Q(X)^2 = 1$
AND ASSOCIATED HYPERELLIPTIC CURVES

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ABSTRACT: The title equations (with $a, b \in \mathbb{Q}$) are connected with Jacobians of hyperelliptic curves $C_{m,a,b} : y^2 = x^{2m} + ax + b$ defined over \mathbb{Q} . More precisely, these equations have a nontrivial solution if and only if the class of the divisor $\infty^+ - \infty^-$ is a torsion point in Jacobian $Jac(C_{m,a,b})$, where ∞^+ and ∞^- are two points at infinity in $C_{m,a,b}$. We show that if $ab = 0$ then the title equations have nontrivial solutions (and we write explicit formulae). On the other hand, we prove that for any $m > 1$ there exists infinitely many pairs (a, b) such that our equations have no nontrivial solutions. Moreover, for $m = 2, 3$ for almost all (a, b) with $ab \neq 0$, these equations have no nontrivial solutions. We also give infinitely many explicit examples when nontrivial solution does not exist.