SOME RESULTS AND PROBLEMS INVOLVING HARDY'S FUNCTION $Z(T)$

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Abstract: Let $Z(t) = \zeta(\frac{1}{2} + it)\chi^{-1/2}(\frac{1}{2} + it)$ denote as usual Hardy's function, where $\zeta(s) = \chi(s)\zeta(1 - s)$ is the functional equation for the Riemann zeta-function $\zeta(s)$. Thus $Z(t) \in \mathbb{R}$ for t real, and $|Z(t)| = |\zeta(\frac{1}{2}+it)|$. It is proved that, for $t \geqslant t_0 > 0$,

$$\max_{T\leqslant t\leqslant T+H, Z(t)>0} Z(t) \gg (\log T)^{1/4} \qquad (T^{\vartheta+\varepsilon} \leqslant H \leqslant T),$$

$$\max_{T\leqslant t\leqslant T+H, Z(t)<0} -Z(t) \gg (\log T)^{1/4} \qquad (T^{\vartheta+\varepsilon} \leqslant H \leqslant T),$$

where $\vartheta = \frac{17}{110} = 0.15\overline{45}$, and a similar result for large values of $Z^{(k)}(t)$, where $k \geqslant 1$ is a fixed integer. Several related topics are also discussed, including the recent result with S.M. Gonek on the distribution of positive and negative values of $Z(t)$. Namely both the subsets of $[T, 2T]$ where $Z(t) > 0$ (resp. $Z(t) < 0$) have measure $\gg T$.

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