

# DERIVATIVES AND FAST EVALUATION OF THE TORNHEIM ZETA FUNCTION

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ABSTRACT: We study analytic properties of the Tornheim zeta function  $\mathcal{W}(r, s, t)$ , which is also named after Mordell and Witten. In particular, we evaluate the function  $\mathcal{W}(s, s, \tau s)$  ( $\tau > 0$ ) at  $s = 0$  and, as our main result, find the derivative of this function at  $s = 0$ , which turns out to be surprisingly simple. These results were first conjectured using high-precision calculations based on an identity due to Crandall that involves a free parameter and provides an analytic continuation. This identity was also the main tool in the eventual proofs of our results. Furthermore, we derive special values of a permutation sum and study an alternating analogue of  $\mathcal{W}(r, s, t)$ . We conclude this talk with some remarks and results on higher-order Tornheim zeta functions and their derivatives.

This is joint work with Jonathan Borwein (1951–2016).